Chapter - 1

IMAGE SAMPLING & QUANTIZATION

Digital Image Processing

IV/IV B.Tech – 1st Sem

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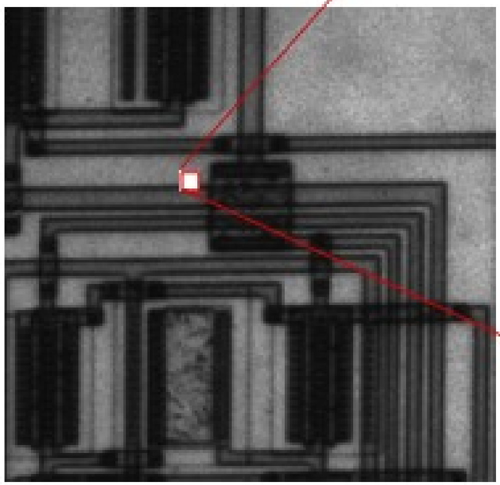
Digital Image Fundamentals

Digital Images and Pixels

- A digital image is the representation of a continuous image *f(x,y)* by a 2-d array of discrete samples.
- The amplitude of each sample is quantized to be represented by a finite number of bits.
- Each element of the 2-d array of samples is called a ***pixel*** or ***pel*** (from "picture element")
- Pixels are point samples, without extent.

A Digital Image is Represented by Numbers

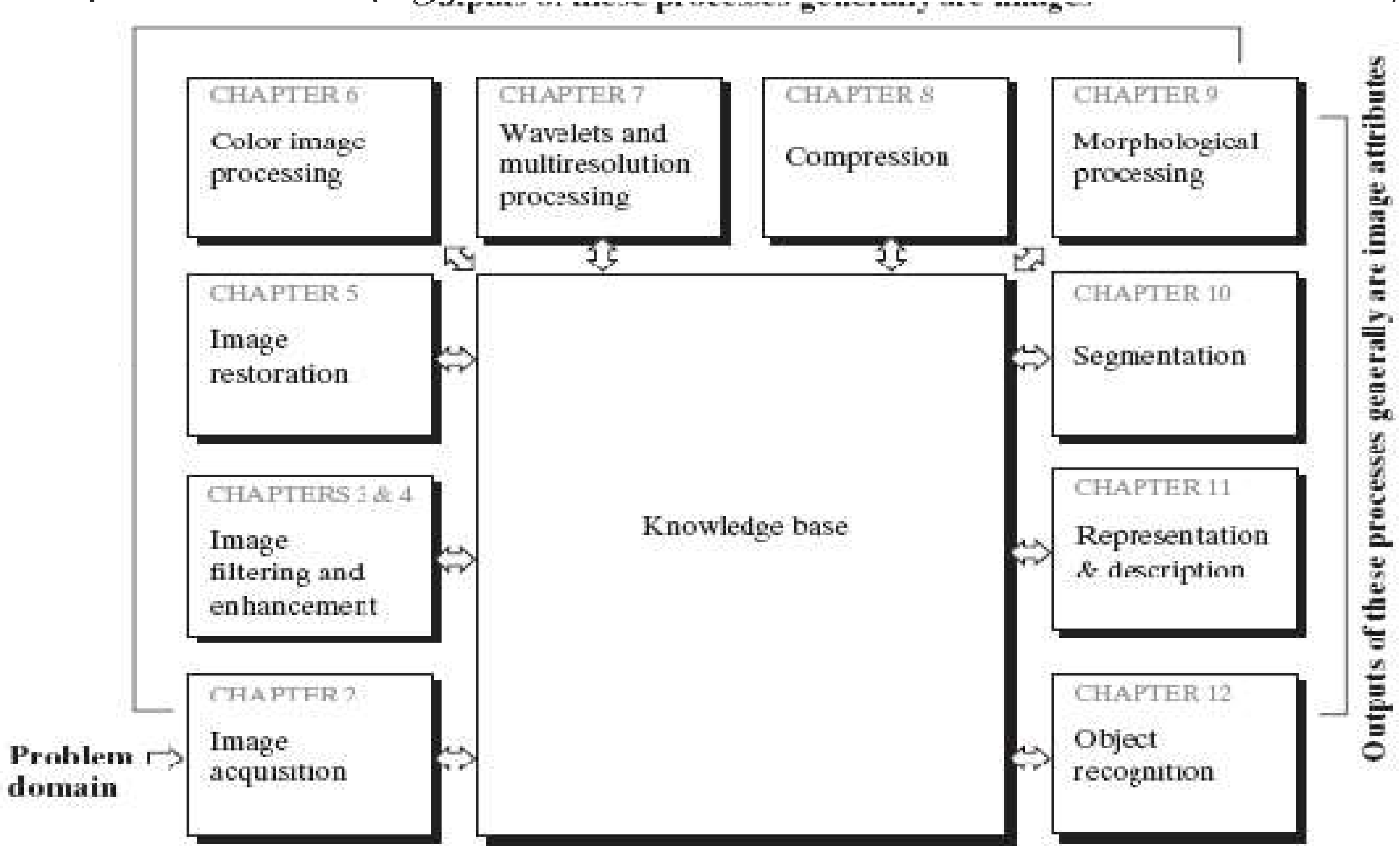


128	125	107	105	110	118	116	114	110
121	122	115	108	106	107	116	116	107
110	114	112	107	105	103	106	106	100
100	96	100	99	94	94	101	101	89
85	82	81	80	76	75	80	82	72
58	58	56	54	53	52	51	49	45
41	41	41	39	39	38	36	35	33
43	43	42	43	41	41	41	43	40
60	60	59	59	60	59	59	58	56

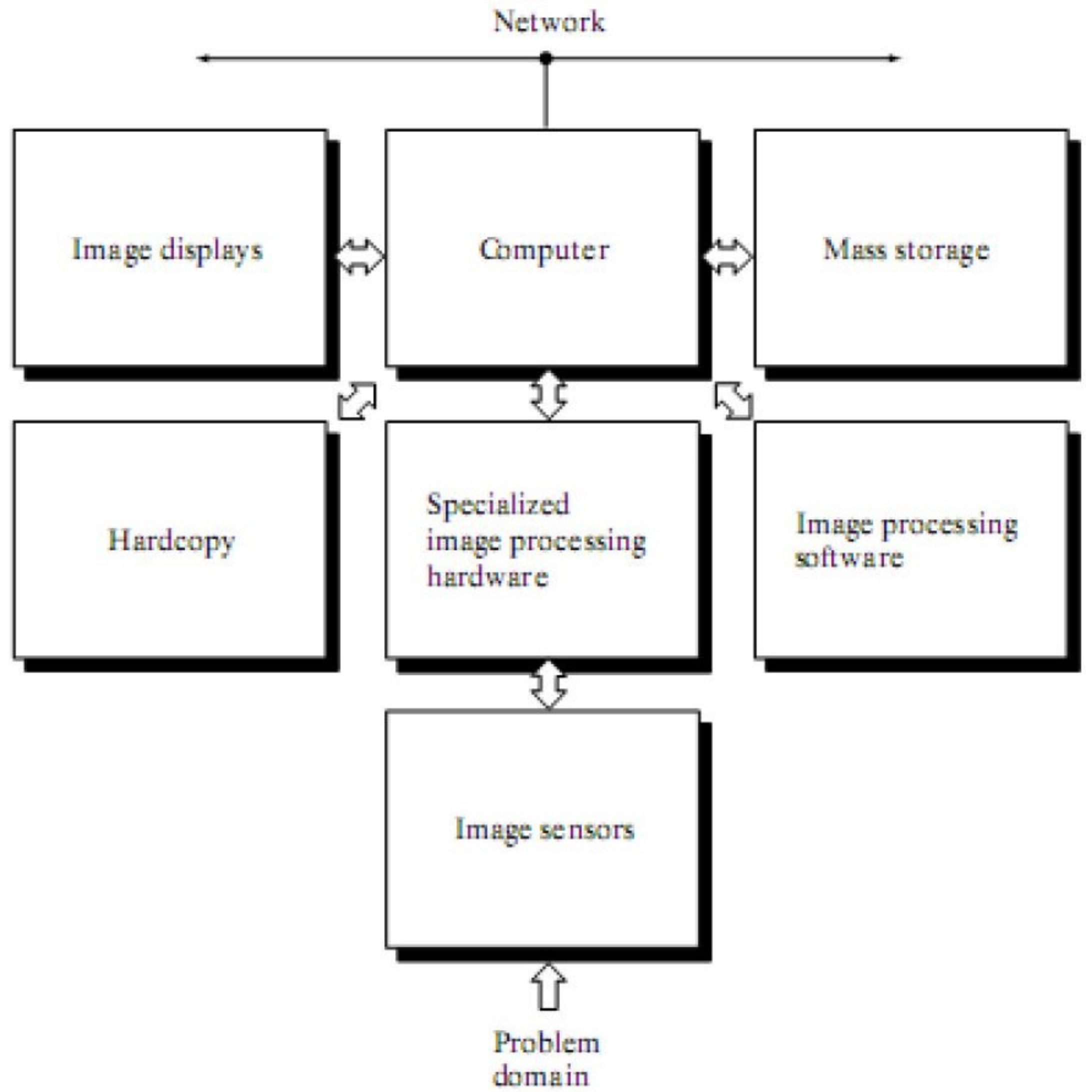
- Pixel = “picture element”
- Represents brightness at one point

A Digital Image can be Represented as a Matrix

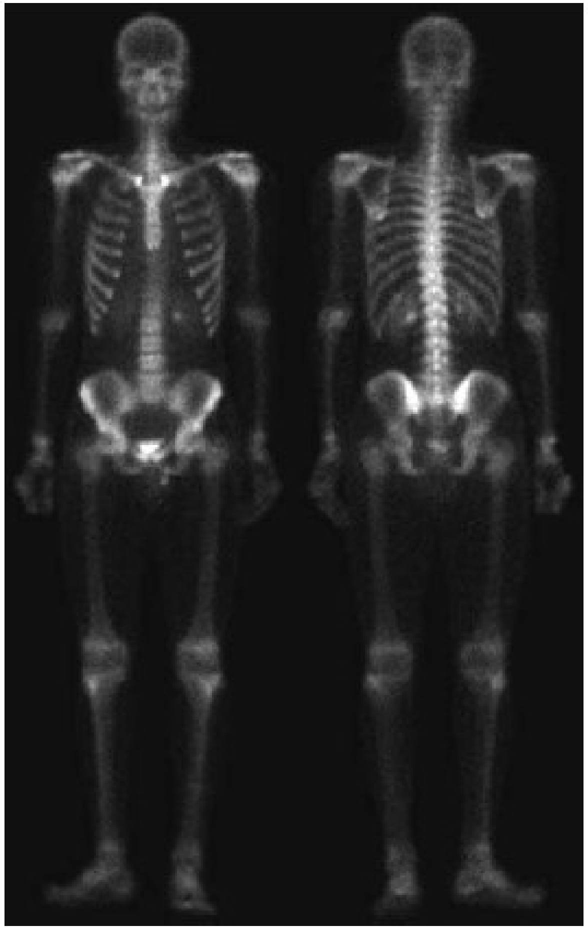
- The pixel values *f(x,y)* are stored into the matrix in "natural" order,
-

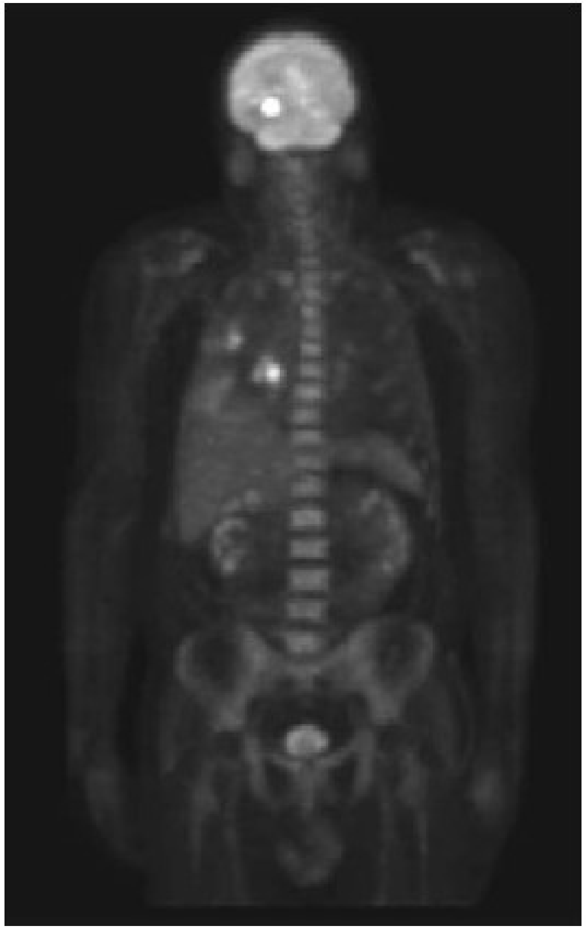


Components of Image Processing System



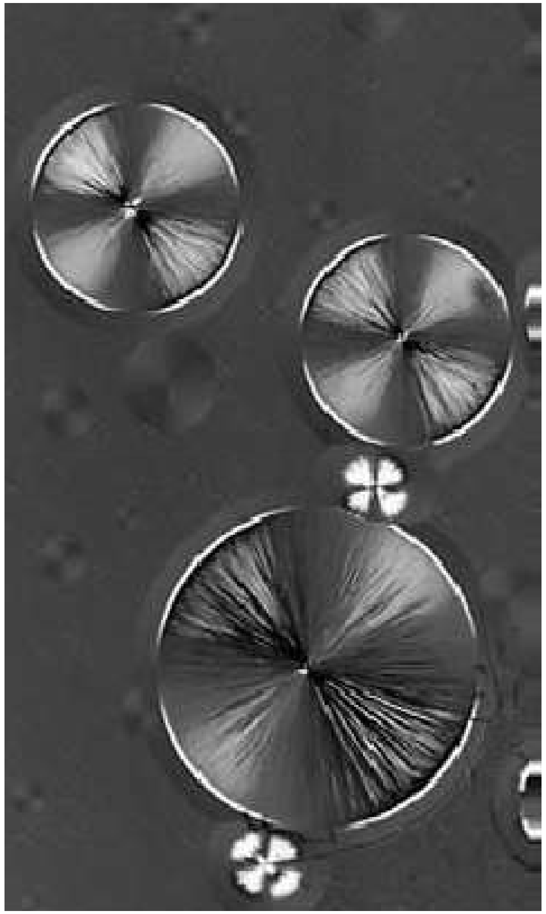
Gamma-Ray Imaging



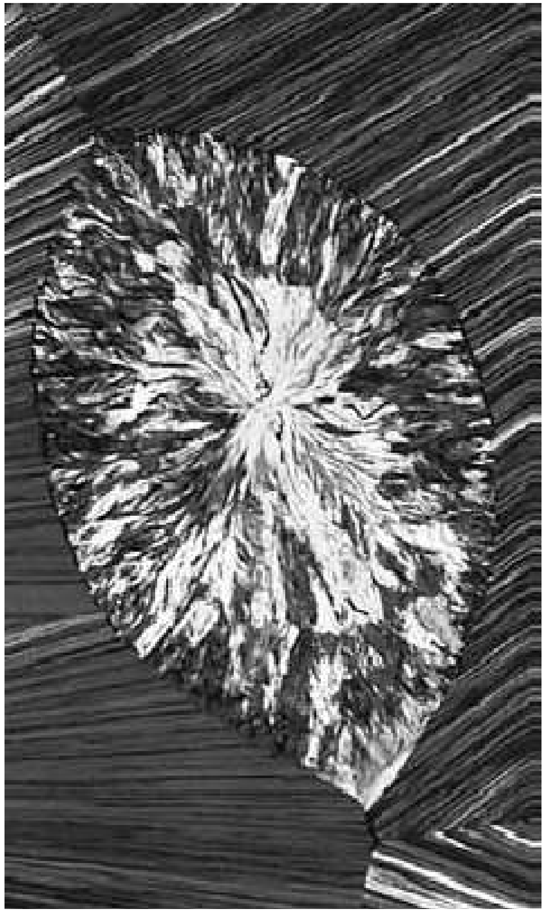


X-Ray Imaging

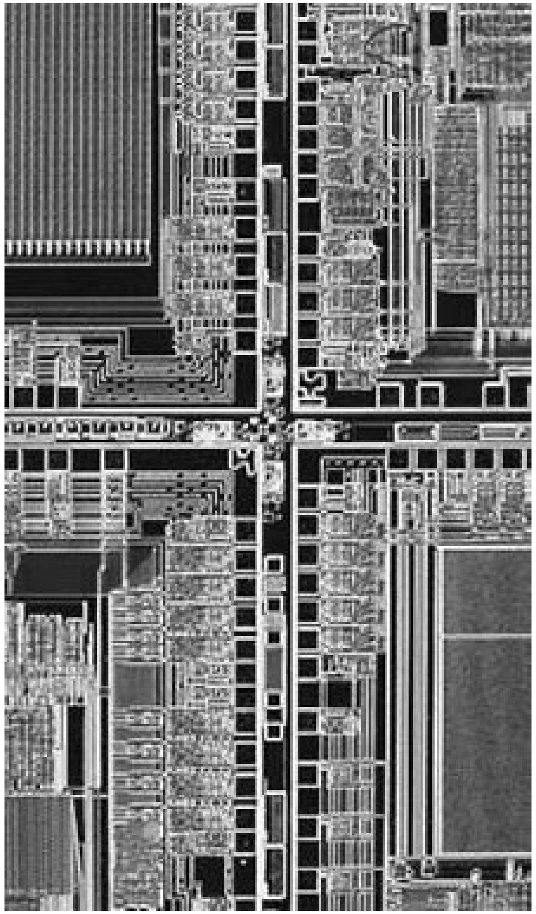
Light microscopy images



Taxol (anticancer agent)



Cholesterol



Microprocessor

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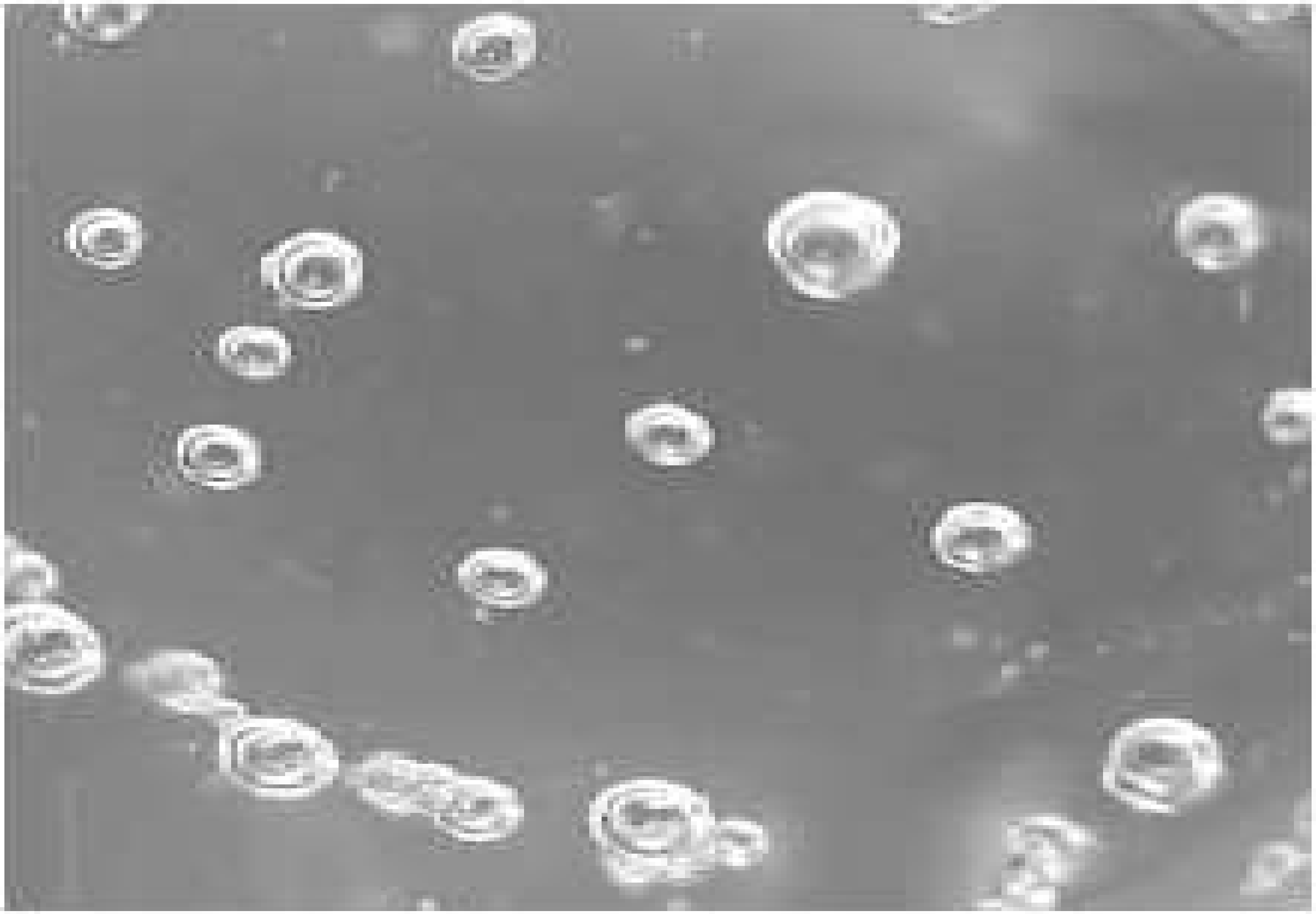
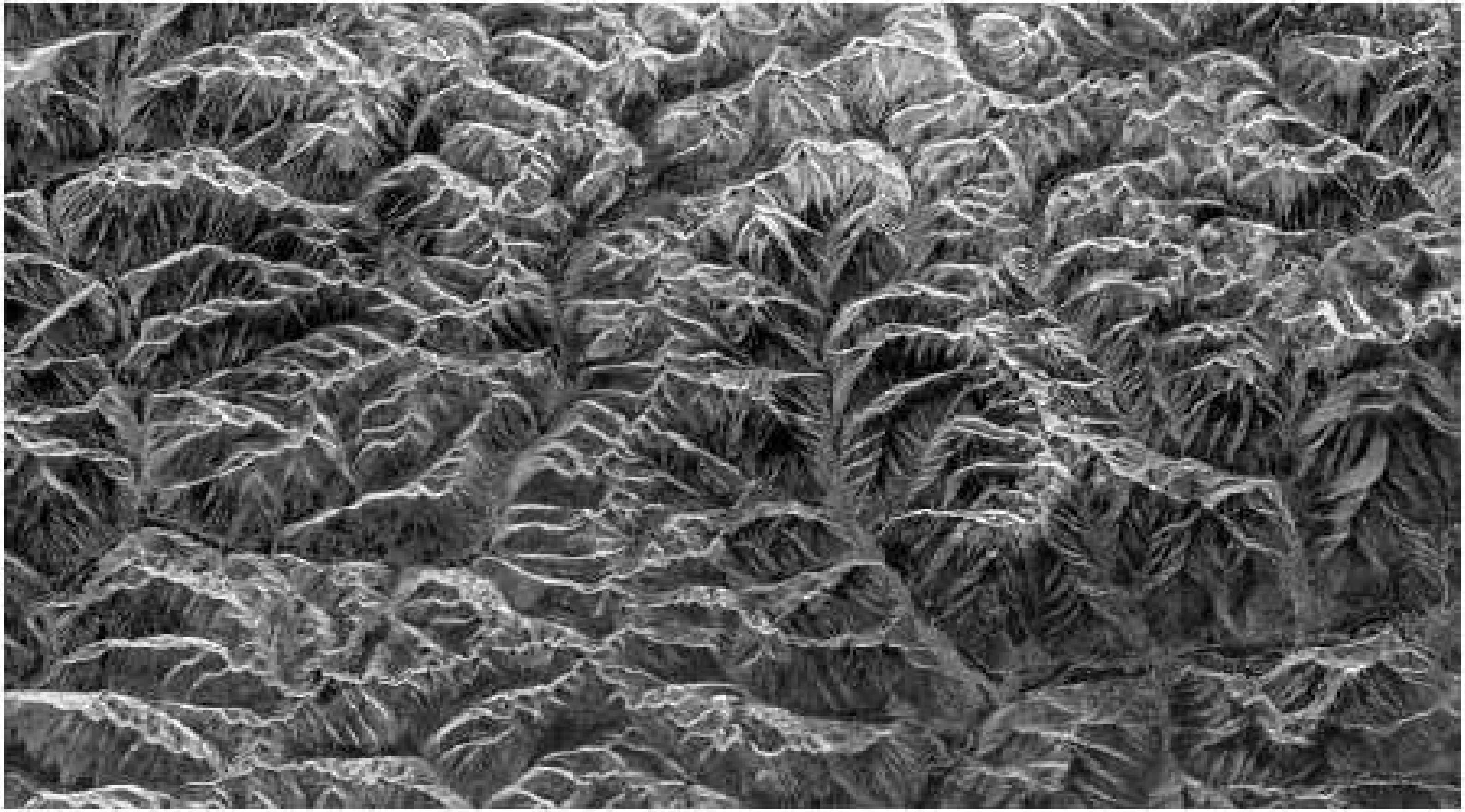
Satellite image of Hurricane Katrina

Air bubbles in a clear-plastic product.

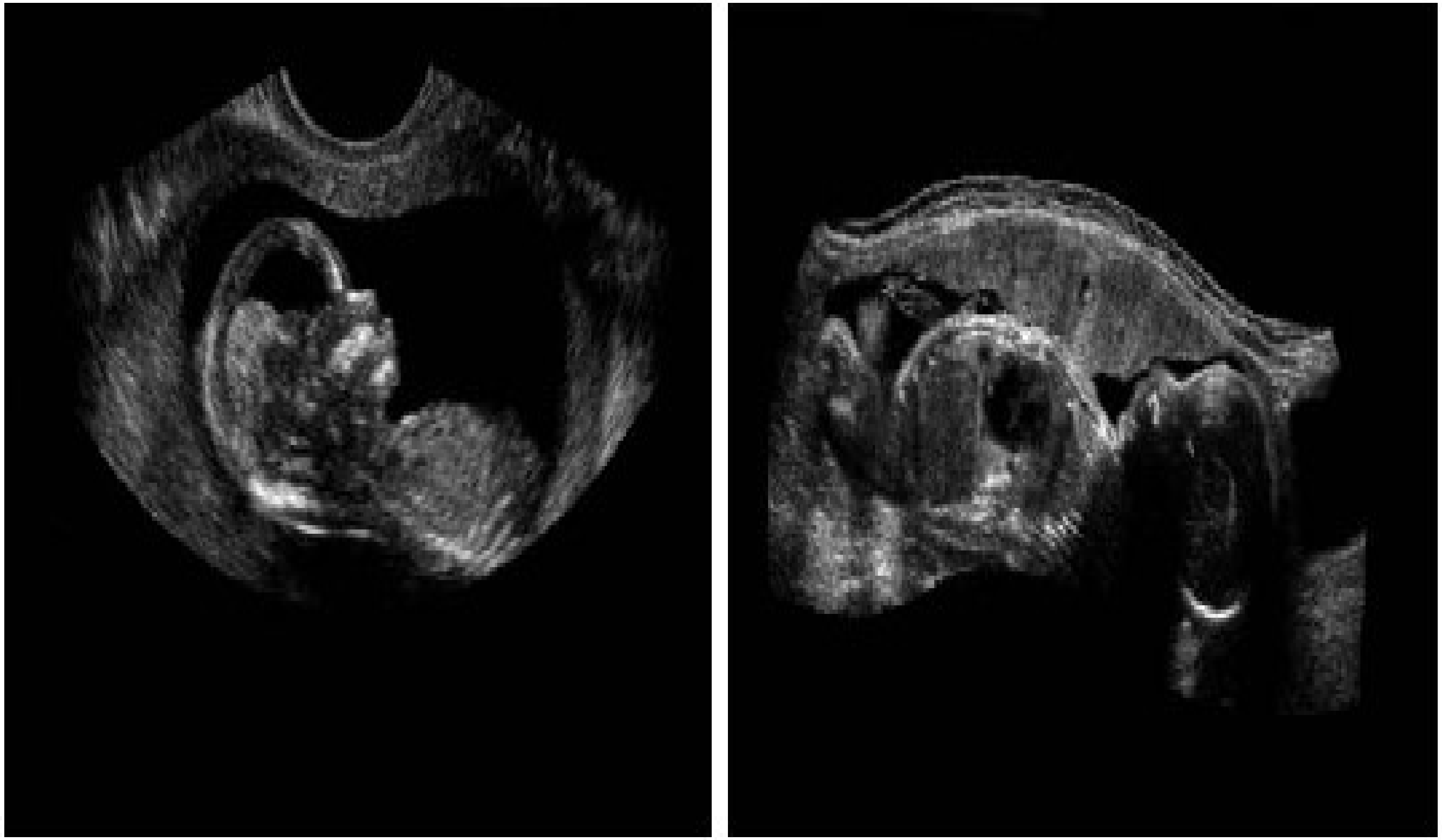


Thumb print

mountains (Southeast Tibet).



Ultrasound imaging (view of Baby).



Simultaneous contrast

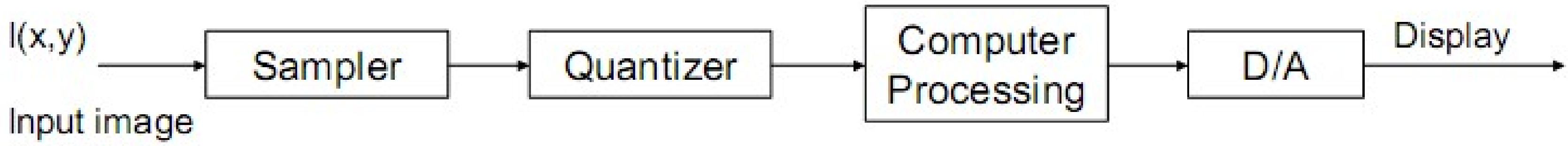
Image Sampling and Quantization

- Digital images are generate from sensed data.
- To create a digital image, we need to convert the continuous sensed data into digital form.
- This involves two processes: *sampling* and *quantization*.
- Digitizing the coordinate values is called *sampling.*
- Digitizing the amplitude values is called *quantization*.

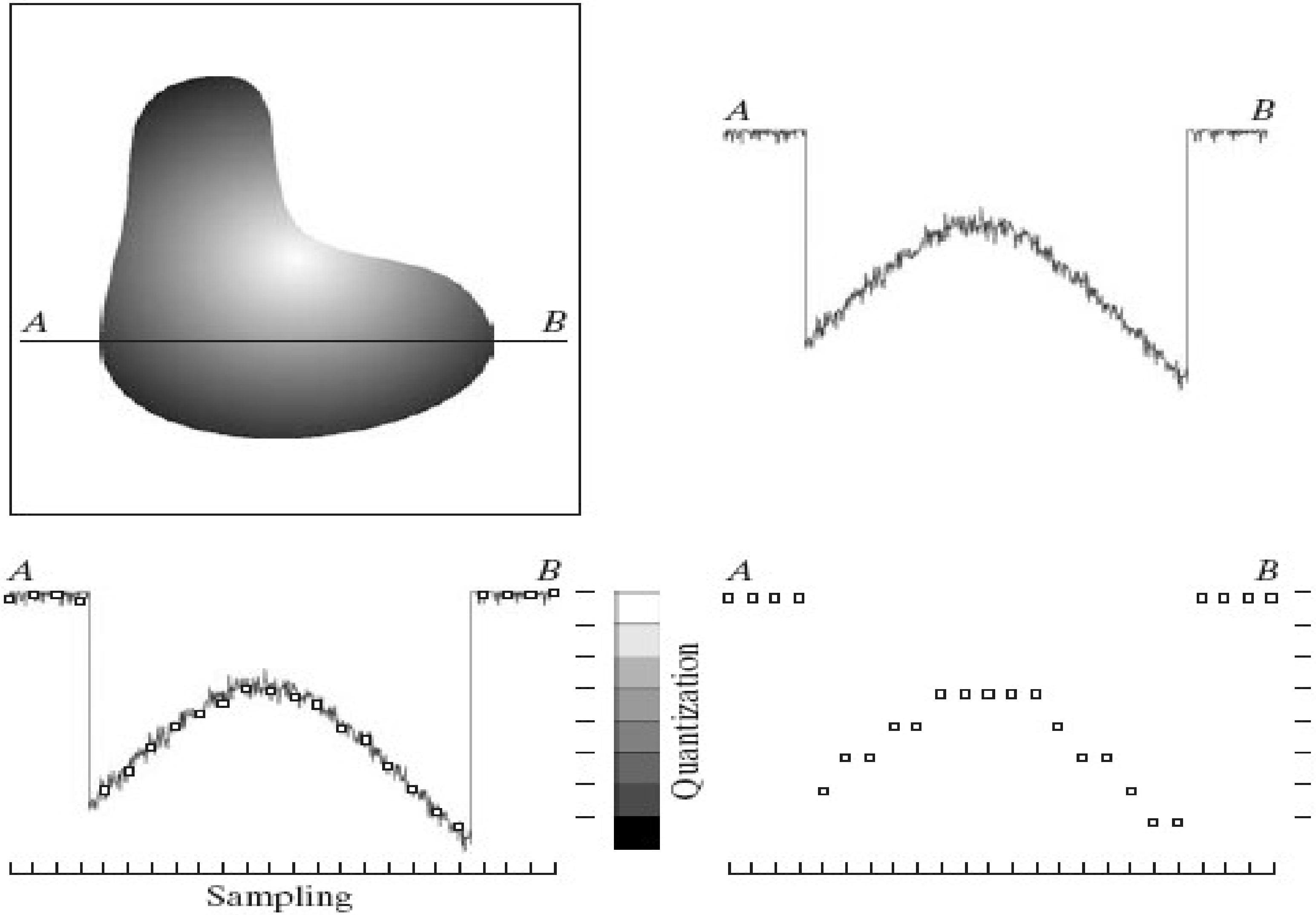
Typical image processing scenario

- Recall: What is an Image?
  - A function f(x,y) over two spatial coordinates of a plane
  - To obtain finitely many data for digital processing
- Sampling (spatially) and Quantizing the luminance values
- Can be done by single chip – Charge-coupled Device (CCD)

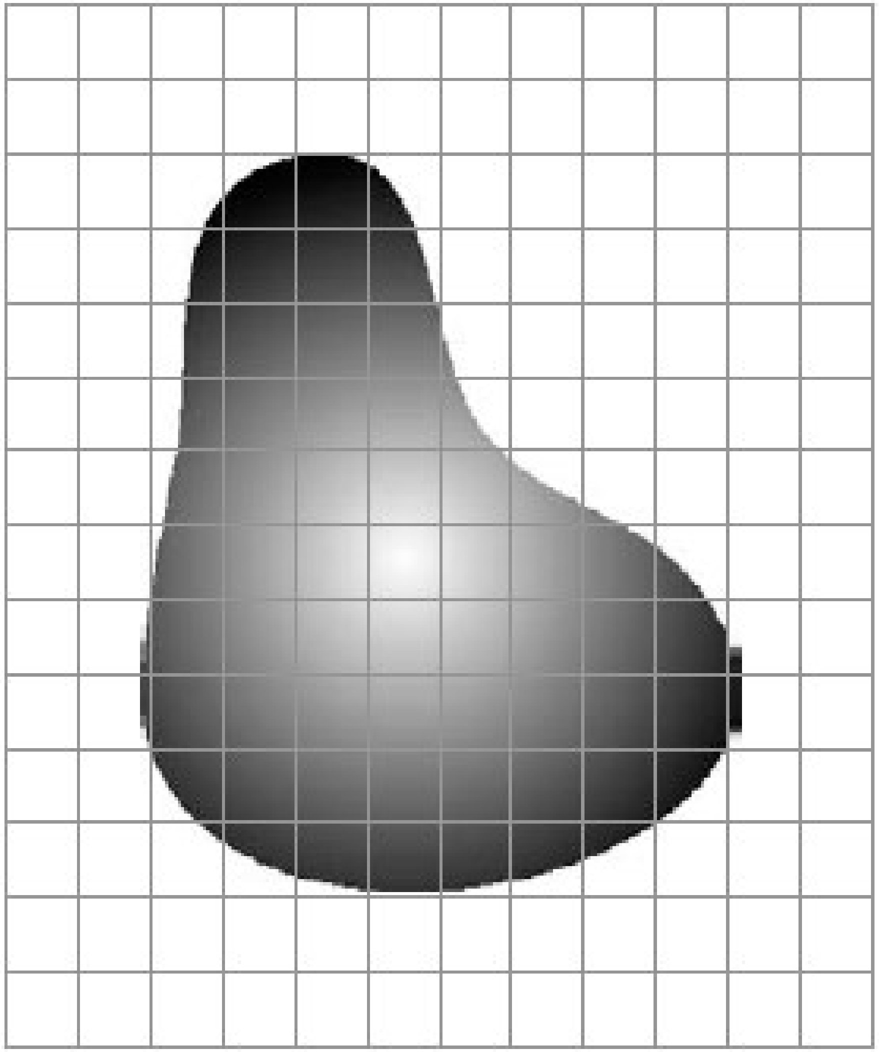
Typical image processing scenario

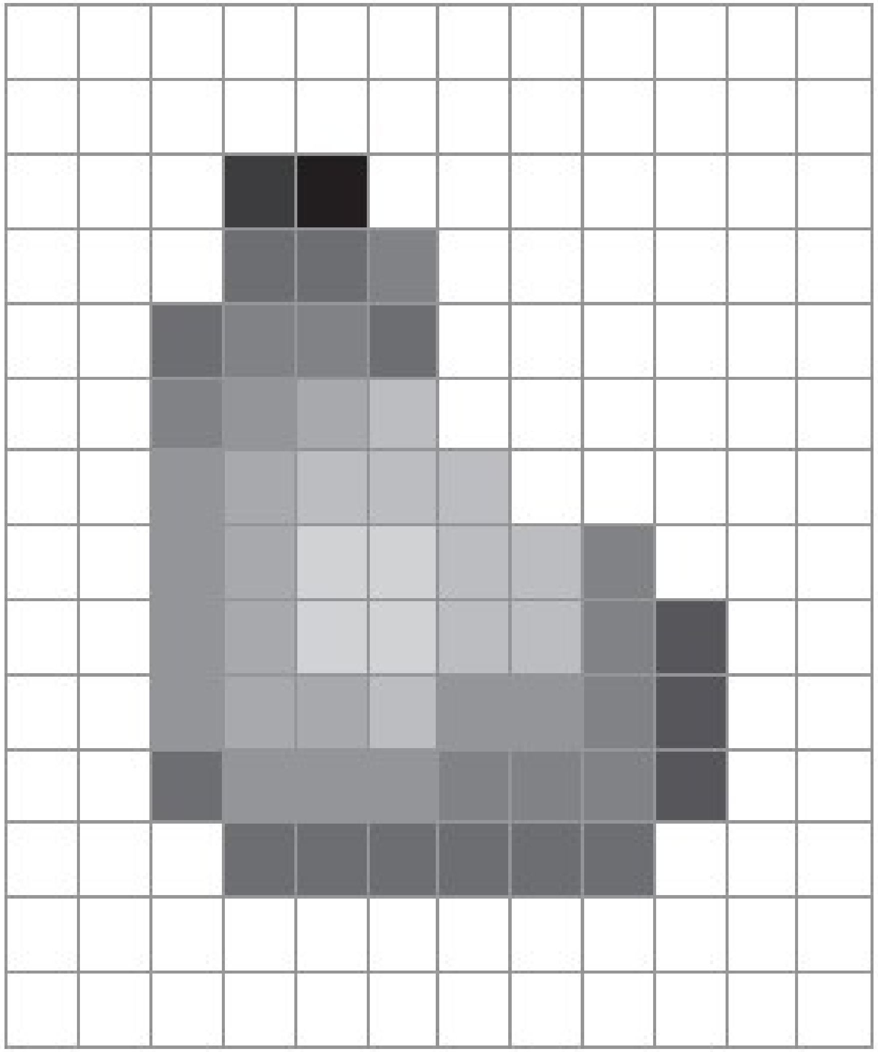


Generating a digital image.



Continuous image and Digital image



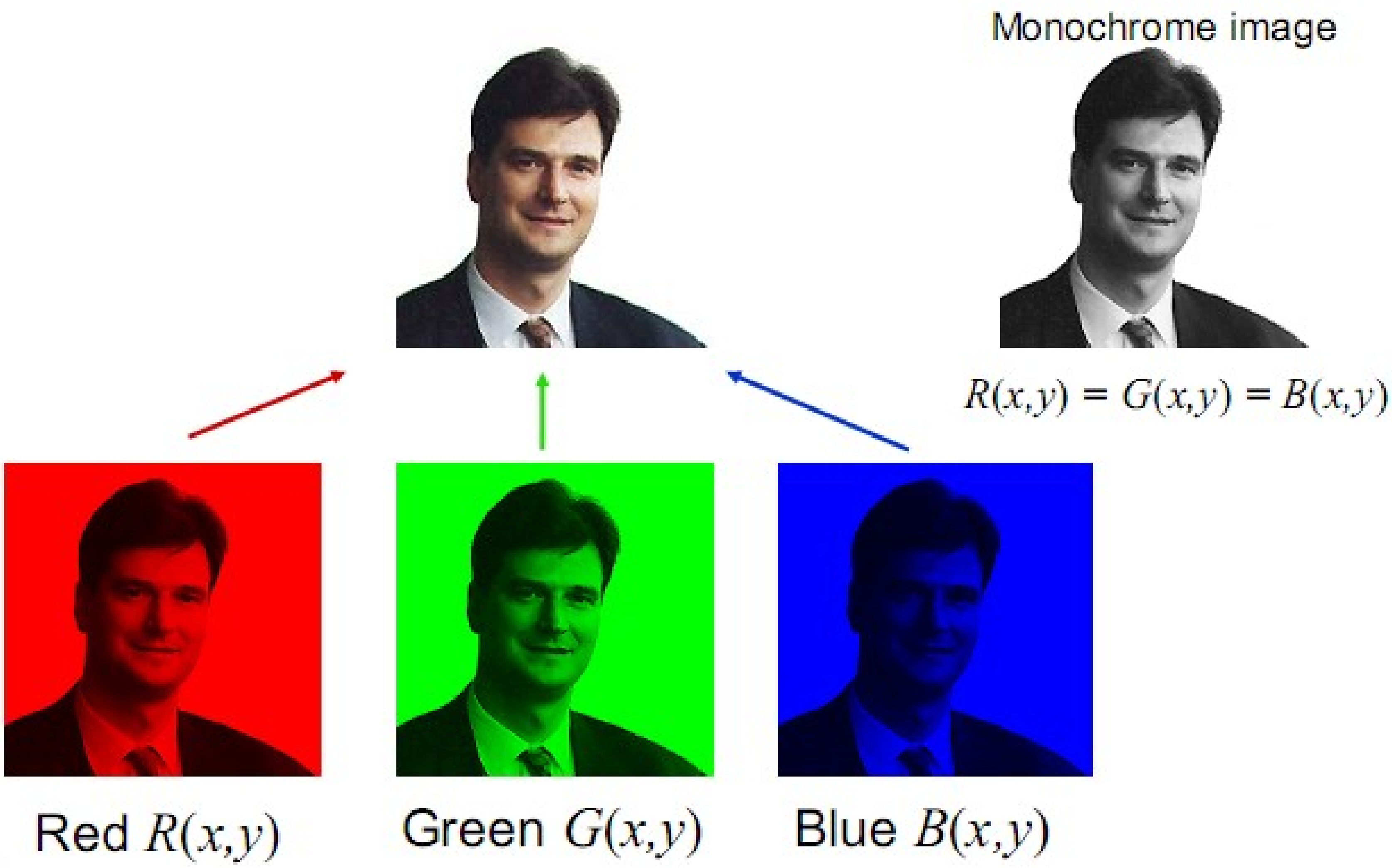


Representing Digital Images

$$f(x,y) = \begin{bmatrix} f(0,0) & f(0,1) & \cdots & f(0,N-1) \\ f(1,0) & f(1,1) & \cdots & f(1,N-1) \\ \vdots & \vdots & & \vdots \\ f(M-1,0) & f(M-1,1) & \cdots & f(M-1,N-1) \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} a_{0,0} & a_{0,1} & \cdots & a_{0,N-1} \\ a_{1,0} & a_{1,1} & \cdots & a_{1,N-1} \\ \vdots & \vdots & & \vdots \\ a_{M-1,0} & a_{M-1,1} & \cdots & a_{M-1,N-1} \end{bmatrix}$$

Digital Image Planes



Storage requirements for digital images

- Image MxN pixels, 2^B gray levels, c color component
  - Size = MxNxBxc
  - Example: M=N=512, B=8, c=1 (i.e., monochrome)
    - Size = 2,097,152 bits (or 256 KByte)
  - Example: MxN=1024x1280, B=8, c=3 (24-bit RGB image)
    - Size = 31,457,280 bits (or 3.75 MByte)

Intensity Levels

Number of storage bits for various values of N and k. L is the number of intensity levels.

N/k	1 (L = 2)	2 (L = 4)	3 (L = 8)	4 (L = 16)	5 (L = 32)	6 (L = 64)	7 (L = 128)	8 (L = 256)
32	1,024	2,048	3,072	4,096	5,120	6,144	7,168	8,192
64	4,096	8,192	12,288	16,384	20,480	24,576	28,672	32,768
128	16,384	32,768	49,152	65,536	81,920	98,304	114,688	131,072
256	65,536	131,072	196,608	262,144	327,680	393,216	458,752	524,288
512	262,144	524,288	786,432	1,048,576	1,310,720	1,572,864	1,835,008	2,097,152
1024	1,048,576	2,097,152	3,145,728	4,194,304	5,242,880	6,291,456	7,340,032	8,388,608
2048	4,194,304	8,388,608	12,582,912	16,777,216	20,971,520	25,165,824	29,369,128	33,554,432
4096	16,777,216	33,554,432	50,331,648	67,108,864	83,886,080	100,663,296	117,440,512	134,217,728
8192	67,108,864	134,217,728	201,326,592	268,435,456	335,544,320	402,653,184	469,762,048	536,870,912

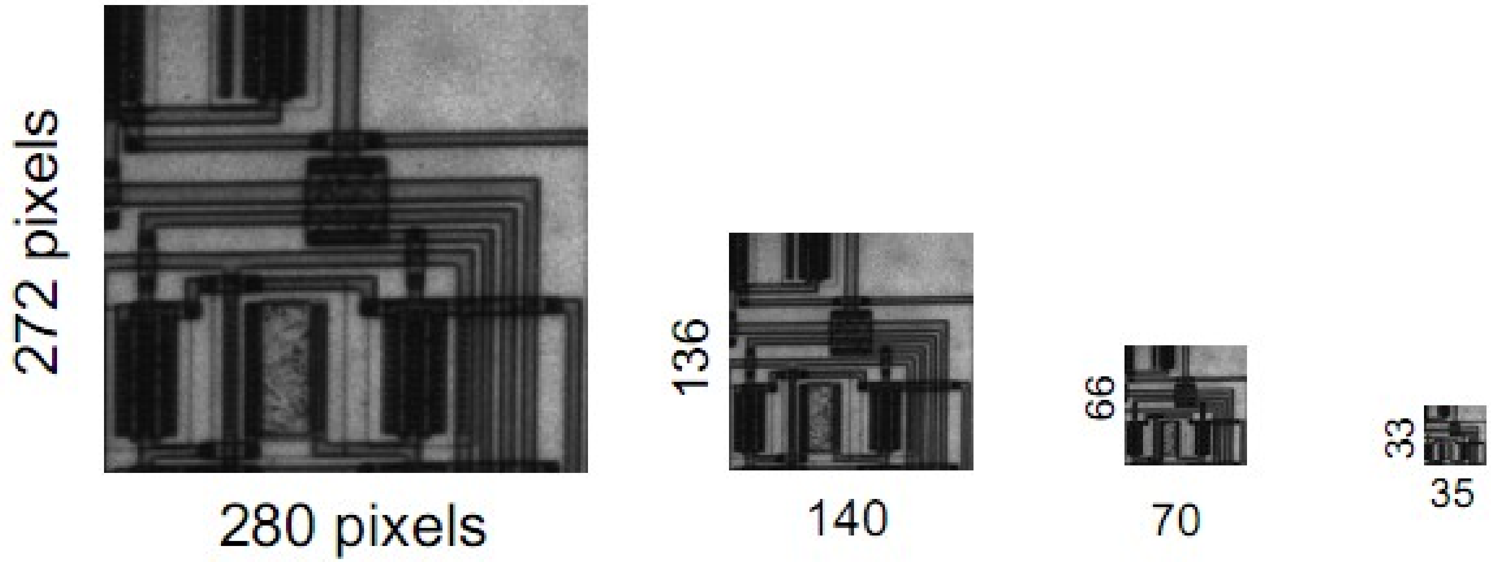
Spatial and Intensity Resolution

- Resolution is the smallest number of discernible line pairs per unit distance.
- Spatial resolution is a measure of the smallest discernible detail in an image.
- For example,100 line pairs per millimeter.
- **Intensity** Resolution is the smallest discernible changes in **Intensity levels**.

Image Size and Resolution

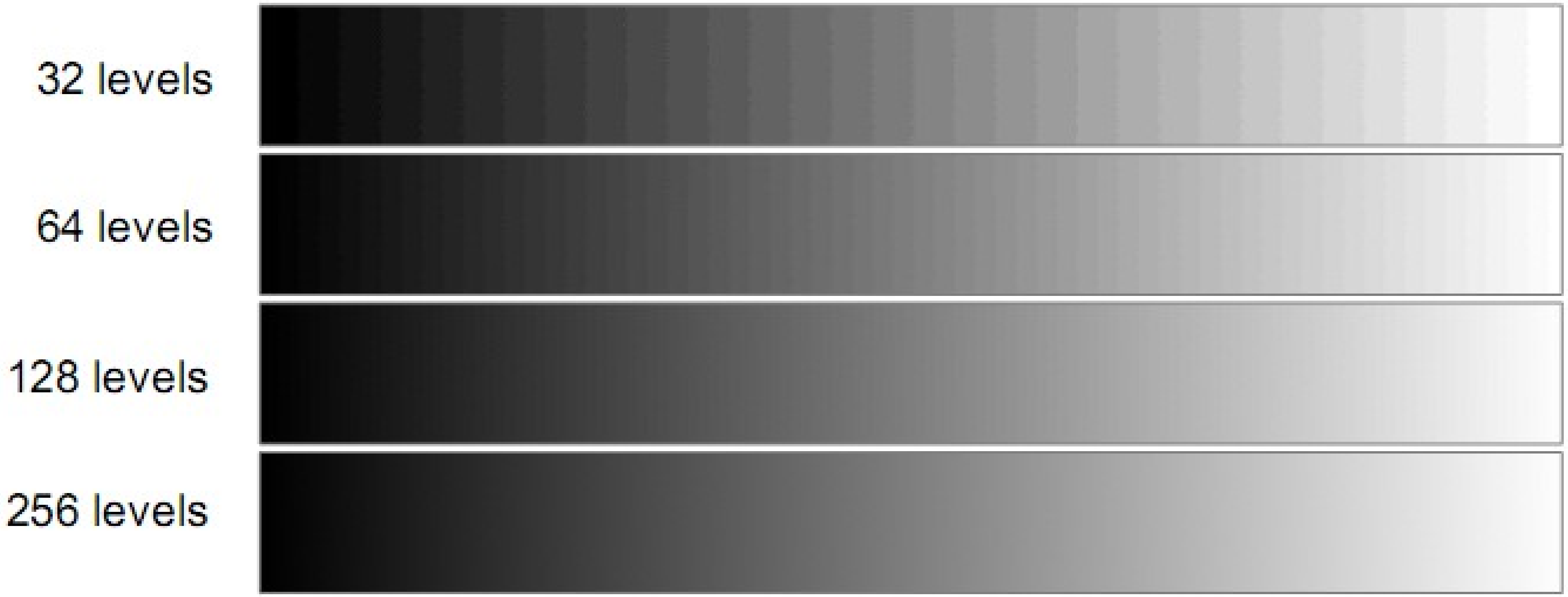
- These images were produced by simply picking every n-th sample horizontally and vertically and replicating that value n x n times.

Images of Different Sizes



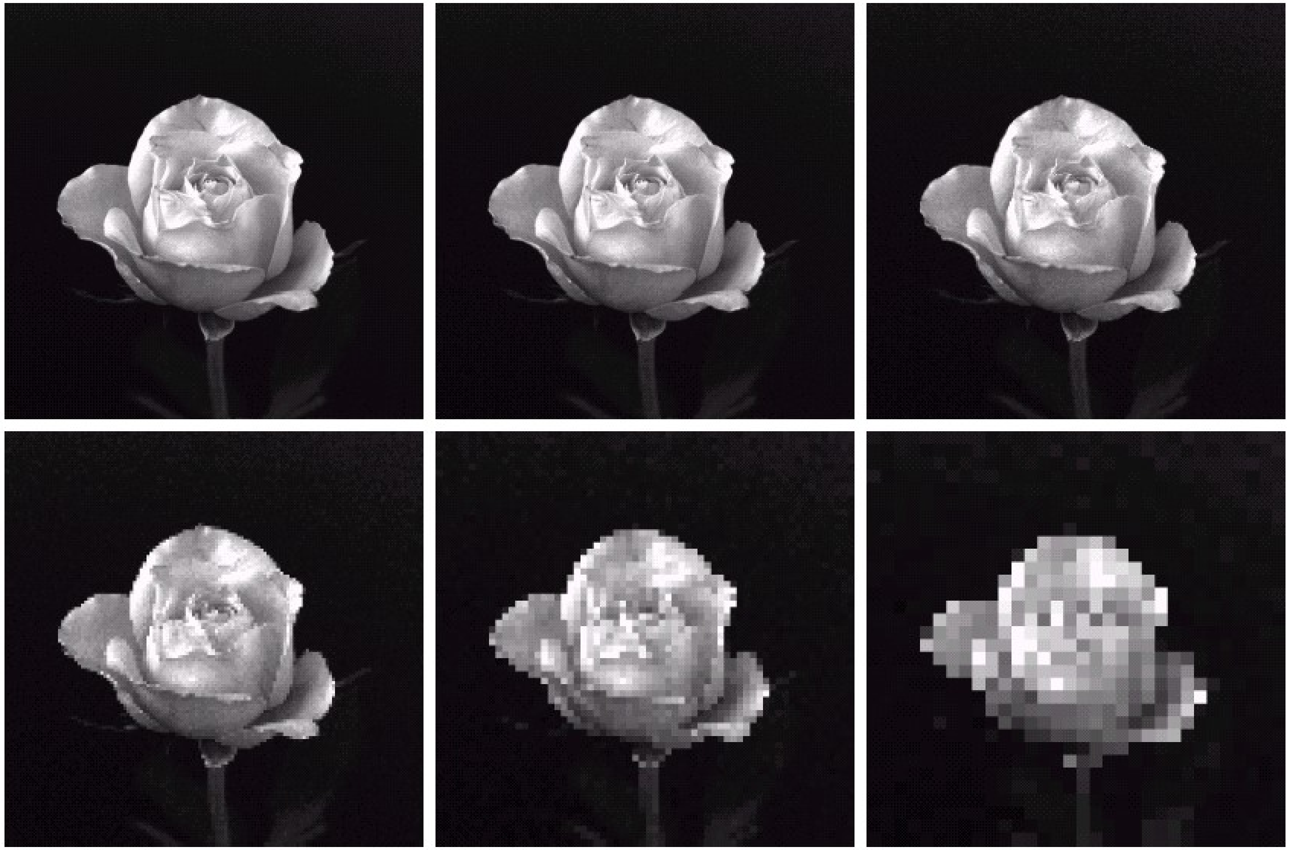
How many gray levels are required?

- Contouring is most visible for a ramp



- Digital images typically are quantized to 256 gray levels

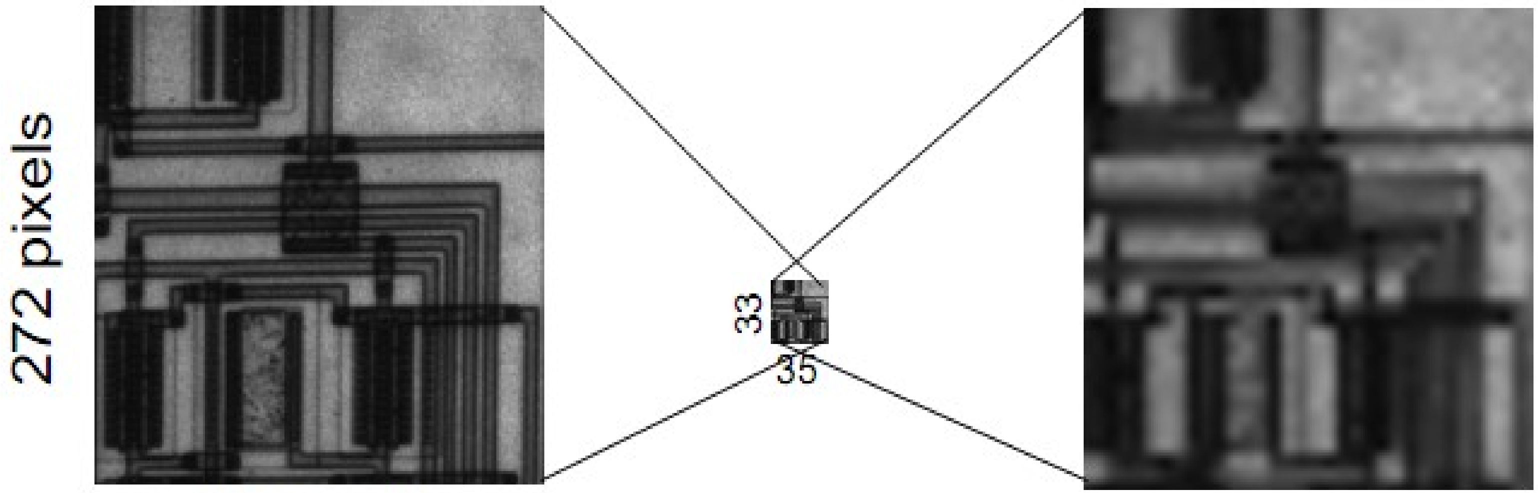
Image Size and Resolution



a b c
d e f

FIGURE 2.20 (a) 1024 × 1024, 8-bit image. (b) 512 × 512 image resampled into 1024 × 1024 pixels by row and column duplication. (c) through (f) 256 × 256, 128 × 128, 64 × 64, and 32 × 32 images resampled into 1024 × 1024 pixels.

Fewer Pixels Mean Lower Spatial Resolution



35 x 33 image interpolated to 280 x 272 pixels

Different numbers of gray levels

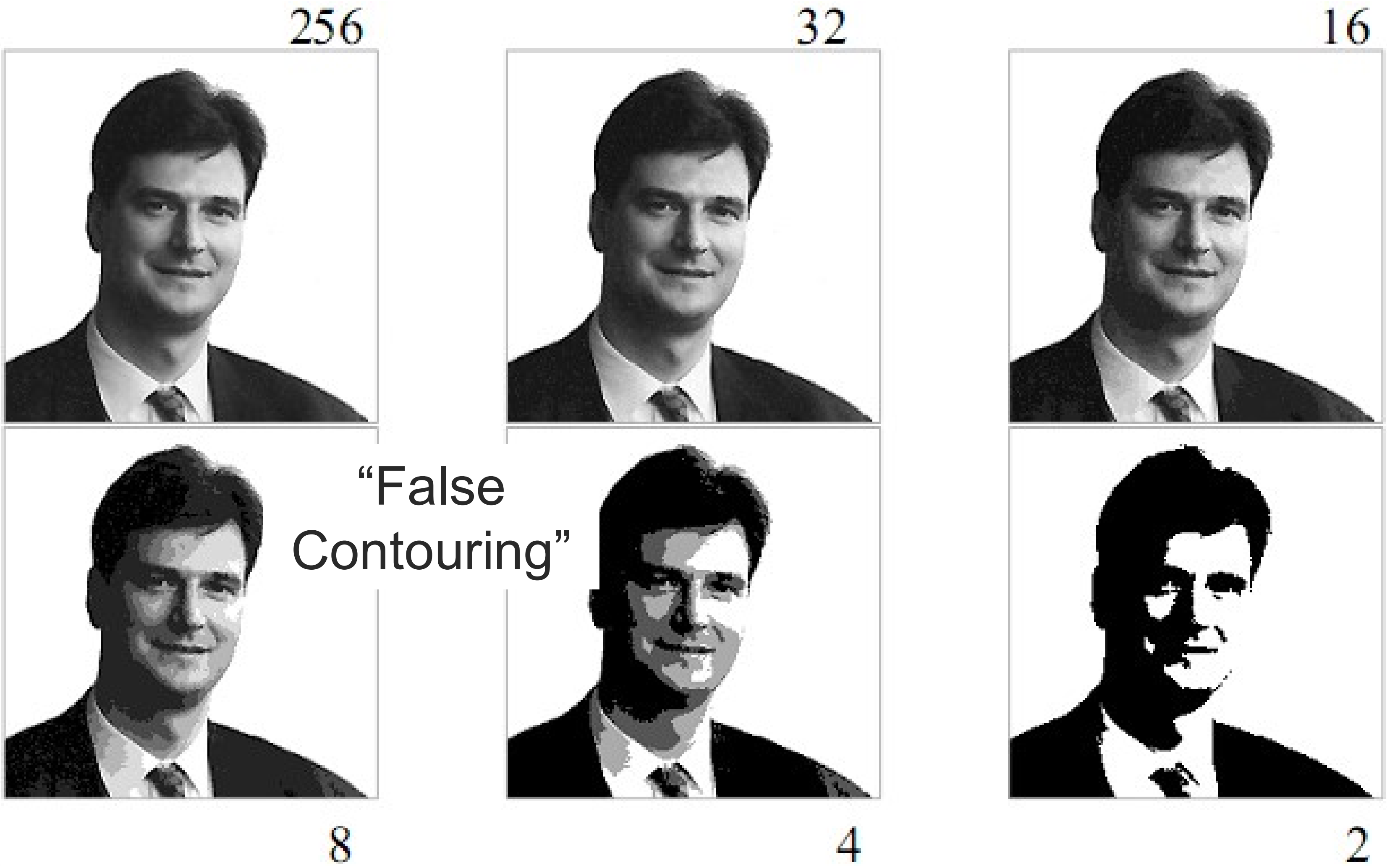


Image Interpolation

- **Interpolation** is the process of using known data to estimate values at unknown locations
- **Interpolation** is a basic tool used extensively in tasks such as zooming, shrinking, rotating, and geometric corrections. which are basically image resampling methods.

Interpolations

- ***Nearest neighbor*** *interpolation ,which* assigns to each new location the intensity of its nearest neighbor in the original image
- ***Bilinear*** *interpolation*, in which we use the four nearest neighbors to estimate the intensity at a given location.

$$v(x, y) = ax + by + cxy + d$$

- ***Bicubic*** *interpolation*, which involves the sixteen nearest neighbors of a point.

$$v(x, y) = \sum_{i=0}^{3} \sum_{j=0}^{3} a_{ij} x^i y^j$$

Its is used in commercial image editing programs, such as Adobe Photoshop and Corel Photopaint.

Effects of reducing spatial resolution

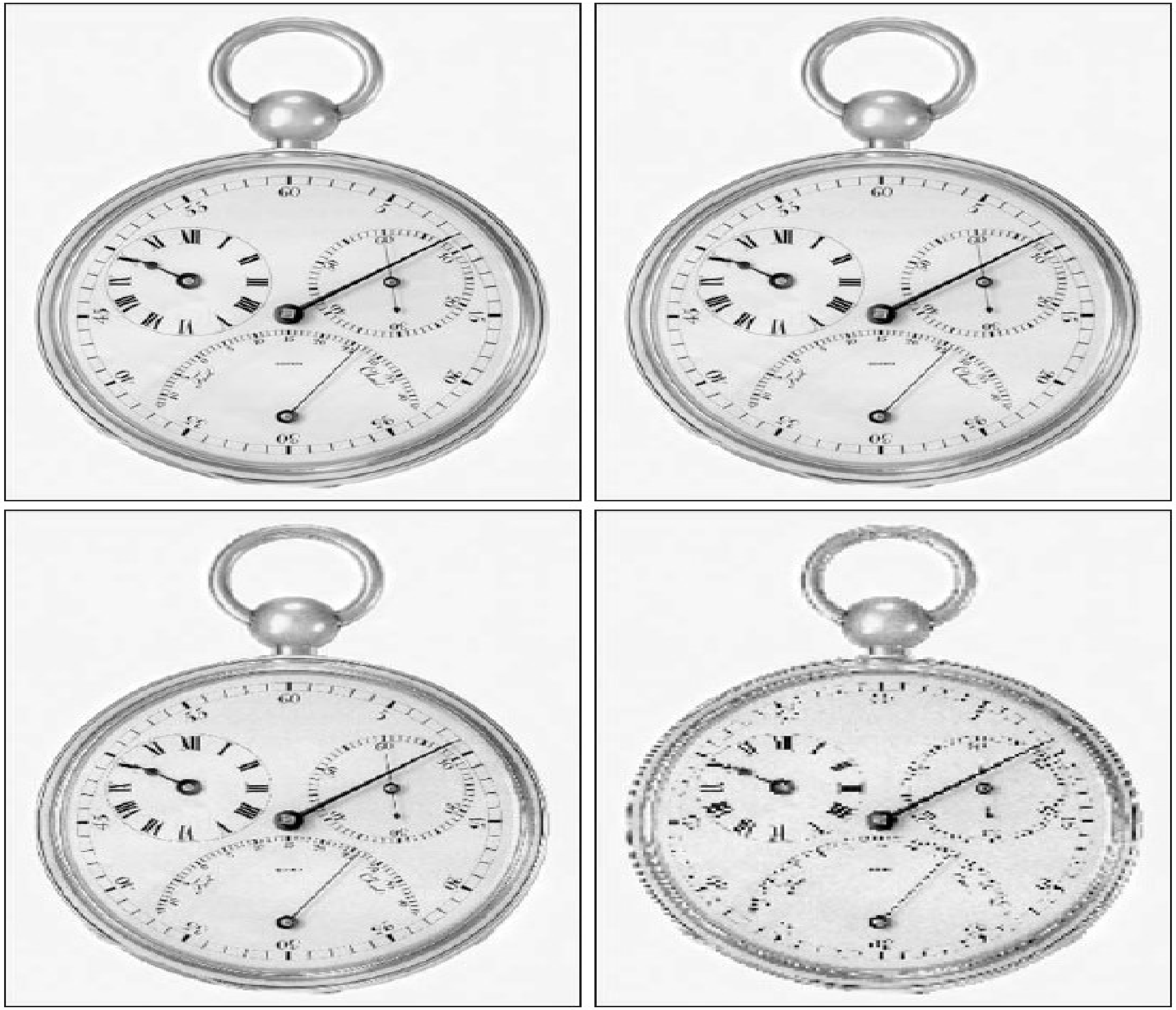


Image reduced to 72 dpi and zoomed back to its original size

Nearest neighbor

Bilinear

Bicubic

Image reduced to 150 dpi and zoomed back to its original size

Nearest neighbor

Bilinear

Bicubic

Basic Relation b/w pixels

- Neighbors of a pixel
- Connectivity, Adjacency
- Labeling of Connected Components
- Distance Measures
- Arithmetic/Logic Operations

Relationships between Pixels

(X-1,Y-1)	(X,Y-1)	(X+1,Y-1)
(X-1,Y)	(X,Y)	(X+1,Y)
(X-1,Y+1)	X,Y+1	(X+1,Y+1)

Relationships between Pixels

(X-1,Y-1)	(X,Y-1)	(X+1,Y-1)
(X-1,Y)	(X,Y)	(X+1,Y)
(X-1,Y+1)	X,Y+1	(X+1,Y+1)

- Horizontal and Vertical Neighbors *($N_4(p)$)*
- Diagonal Neighbors *($N_D(p)$)*
- Both are *($N_8(p)$)*

Neighbors of a pixel

- a pixel p at coordinate (x,y) has
  - $N_4(p)$: 4-neighbors of p

    (x+1, y), (x-1,y),(x,y+1), (x,y-1)

```
     x
  x  p  x
     x
```

- $N_D(p)$: 4-diagonal neighbors of p

  (x+1, y+1), (x+1,y-1),(x-1,y+1), (x-1,y-1)

```
  x     x
     p
  x     x
```

- $N_8(p)$: 8-neighbors of p :

  a combination of $N_4(p)$ and $N_D(p)$

```
  x  x  x
  x  p  x
  x  x  x
```

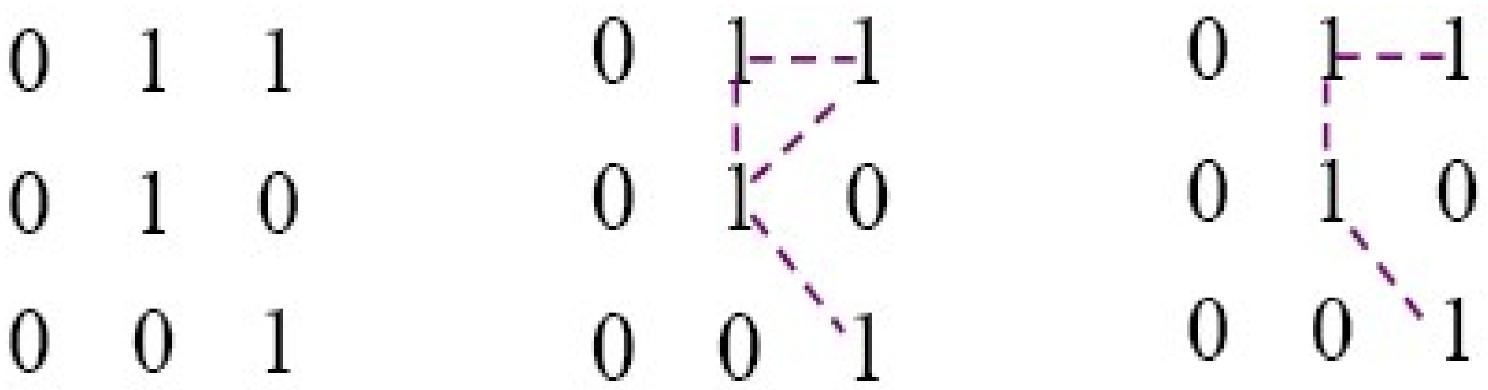
Connectivity

- Connectivity between pixels is used in establishing boundaries of objects and components of regions in an image
- Two pixels are connected if
  - They are neighbors (i.e. adjacent in some sense -- e.g. $N_4(p)$, $N_8(p)$, …)
  - Their gray levels satisfy a specified criterion of similarity (e.g. equality, …)
- V is the set of gray-level values used to define adjacency (e.g. V={1} for adjacency of pixels of value 1

Adjacency

- Let V be the set of gray-level values used to defined adjacency
  - 4-adjacency :
    - 2 pixels p and q with values from V are 4-connected if q is in the set $N_4(p)$
  - 8-adjacency
    - 2 pixels p and q with values from V are 8-connected if q is in the set $N_8(p)$
  - m-adjacency (mixed adjacency)
    - 2 pixels p and q with values from V are m-adjacency if
      - q is in the set $N_4(p)$ or
      - q is in the set $N_D(p)$ and the set $N_4(p) \cap N_4(q)$ is empty.
      - (the set of pixels that are 4-neighbors of both p and q whose values are from V)

Example



- m-connectivity eliminates the multiple path connections that arise in 8-connectivity.
- Two pixels are said to be adjacency if they are connected.

Labeling of connected components

- scan the image from left to right
- Let p denote the pixel at any step in the scanning process.
- Let r denote the upper neighbor of p.
- Let t denote the left-hand neighbors of p, respectively.
- when we get to p, points r and t have already been encountered and labeled if they were 1's.

	r
t	p

Labeling of connected components

- if the value of p = 0, move on.
- if the value of p = 1, examine r and t.
  - if they are both 0, assign a new label to p.
  - if they are both 1
    - if they have the same label, assign that label to p.
    - if not, assign one of the labels to p and make a note that the two labels are equivalent. (r and t are connected through p).
- at the end of the scan, all points with value 1 have been labeled.
- do a second scan, assign a new label for each equivalent labels.

Path

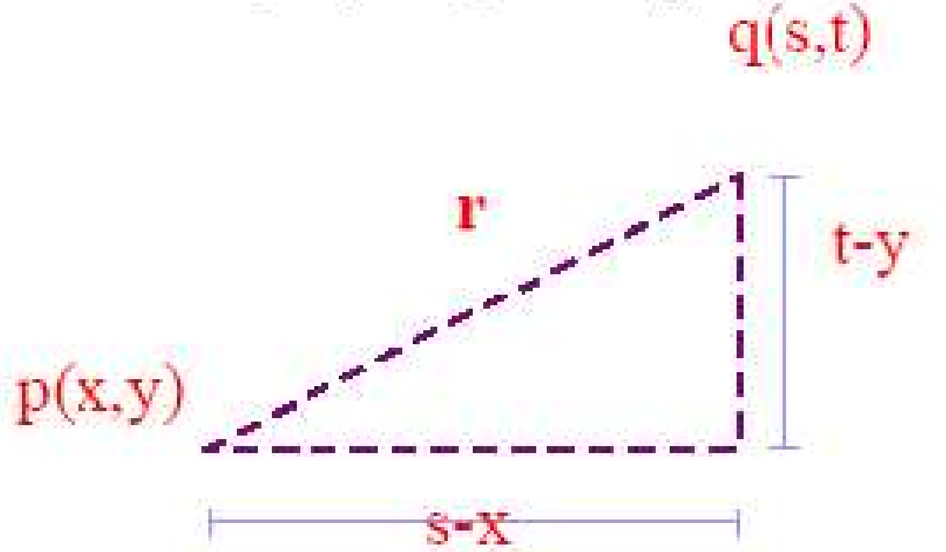
- a path from pixel p with coordinates (x,y) to pixel q with coordinates (s,t) is a sequence of distinct pixels with coordinates $(x_0,y_0),(x_1,y_1),\ldots(x_n,y_n)$
  where $(x_0,y_0) = (x,y)$, $(x_n,y_n) = (s,t)$ and
  (x_i,y_i) is adjacent to (xi-1,yi-1)
- n is the length of the path
- we can define 4-,8-, or m-paths depending on type of adjacency specified.

Distance Measures

- for pixel p, q and z with coordinates (x,y), (s,t) and (u,v) respectively,
- D is a distance function or metric if
  - (a) $D(p,q) \geq 0$; $D(p,q) = 0$ iff $D=q$
  - (b) $D(p,q) = D(q,p)$
  - (c) $D(p,z) \leq D(p,q) + D(q,z)$

Euclidean distance between the p and q defined as

$$D_e(p,q) = \left[(x-s)^2 + (y-t)^2\right]^{1/2}$$



radius (r) centered at (x,y)

D_4 distance (city-block distance) between p and q is defined as

$$D_4(p,q) = |x-s| + |y-t|$$

- forms a diamond centered at (x,y)
- e.g. pixels with $D_4 \leq 2$ from p

		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

$D_4 = 1$ are the 4-neighbors of p

- The D_8distance(chessboard distance) between p and q is defined as

$D_8(p,q) = \max(|x-s|,|y-t|)$

Forms a square centered at p

e.g. pixels with $D_8 \leq 2$ from p

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

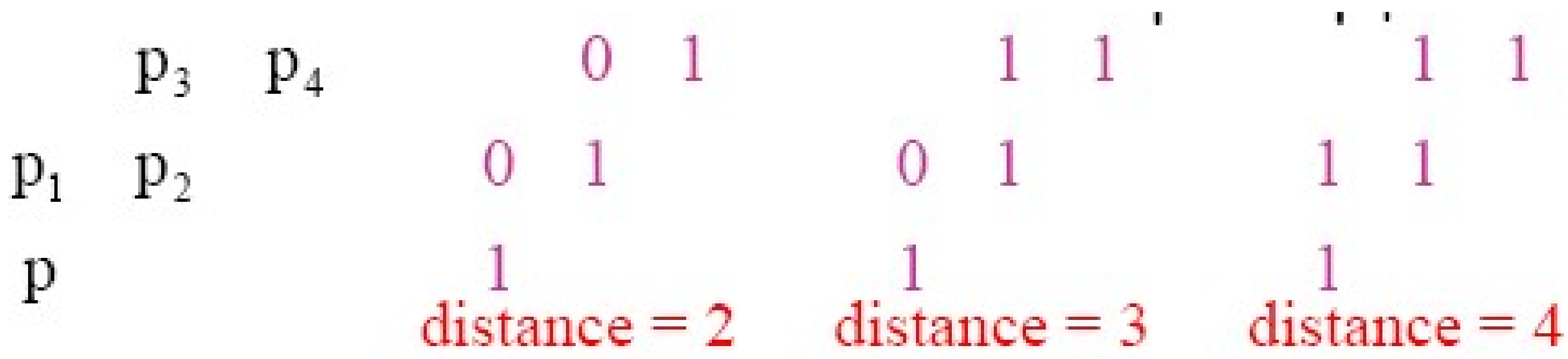
$D_8 = 1$ are the 8-neighbors of p

D_4 and D_8 distances

- D4 distance and D8 distance between pixel p and q = length 4- and 8-path between those 2 points, respectively.
- we can consider both D4 and D8 distances b/w p and q regardless of whether a connected path exists between them because the definitions of these distances involve only the coordinates.

m-connectivity’s distance

- distances of m-connectivity of the path between 2 pixels depends on values of pixels along the path.
- e.g., if only connectivity of pixels valued 1 is allowed. find the m-distance b/w p and p4



Adjacency

- Let *V* be the set of graylevel values used to define adjacency.
- In a binary image , $V=\{1\}$ if we are referring to adjacency of the pixels with value 1.The idea is same for gray scale image,but the *V* typically contains more elements.[0 to 255],set *V* could be any subset of the these 256 values.

- 4-adjacency:-Two pixels *p* and *q* with values from *V* are 4-adjacent if q is in the set N_4(p).
- 8-adjacency:-Two pixels *p* and *q* with values from *V* are 8-adjacent if *q* is in the set N8(p).
- m-adjacency(mixed adjacency):-Two pixels *p* and *q* with values from *V* are m-adjacent if
- (i) *q* is in *N4(p),*or
- (ii) *q* is *ND(p)* and set *N4(p)*and *N4(q)* has no pixels whose values are from *V*

Connectivity & Boundary

- Let ***S*** represent a subset of pixels in an image.Two pixels *p* and *q* are said to be connected in ***S*** if there exists a path between them consisting entirely of pixels in S.
- For any pixels *p* in ***S***, the set of pixels that are connected to it in ***S*** is called a connected component of ***S***.
- Let *R* be subset of pixels in the image.
- We call *R* a region of the image if *R* is connected in the set.
- This extra definition is required because an image has no neighbors beyond its border.

Which Noise?